suggests that the external pressure gradient might be the cause of the time-dependent nature of the bursting phenomenon.

The circulation around the leading-edge vortex was measured along the vortex axis during the whole cycle. The value of the circulation increases linearly with the distance from the apex and follows the pattern of the variation of the freestream velocity. These variations do not depend on whether the vortex bursting takes place or not. In other words, the vorticity shedding from the leading edge is not affected by the breakdown.

## Acknowledgment

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# Boundary Diffusion Controlled Creep Model in Nanocrystals

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#### Introduction

ANOCRYSTALLINE materials (NCMs) by definition are single or multiphase polycrystals with a crystal size on the order of 1–10 nm. NCMs seem to permit the alloying of components, which results in alloys with properties attractive to the aerospace industry. The research around nanocrystals adds a few more difficulties. There is a smaller size and very random arrangement of atoms. Such uncontrollable characteristics are related to the viscosity model by Herring. In aerospace research much time has been spent on changing the shape of crystals by moving atoms around by diffusion. This research has revolved around polycrystalline creep and specifically the work model by Coble.<sup>2</sup>

The focus of this paper is to improve the creep model accepted today along with its diffusivity parameters. By using both the

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Coble and Herring models and Coffey's assumptions<sup>3</sup> for grain boundary sources and by adjusting the vacancy concentration gradient, one can derive a more accurate nanocrystalline creep equation.

## **Background**

The diffusional creep was shown by Herring<sup>1</sup> to result from a change in equilibrium stress-vacancy concentration. The change in vacancy concentrations is

$$\Delta C = C_o \sigma \Omega / kT \tag{1}$$

where  $C_o$  is defined as the equilibrium vacancy concentration at temperature T in a stress-free crystal;  $\sigma$  is the normal to the boundary local stress; k is constant at temperature T; and  $\Omega$  is the vacancy volume (usually denoted as  $\Omega = a_0^3$ ).

In tensile tests by Nieman et al.,<sup>4</sup> when comparisons between nanocrystalline lead (Pb) and coarse grained Pb were made, the former exhibited an increase in hardness and a significant flaw size sensitivity. Once uniaxial tensile stress is applied, the maximum change in vacancy concentration  $\Delta C$  occurs on boundaries perpendicular to the applied stress. When boundaries are parallel to stress then the equilibrium vacancy concentration is  $C_o$ .

Coble, making some assumptions about the vacancy source on a spherical surface, introduced the boundary diffusion creep rate as

$$\dot{e} = 150 \, \sigma D_b W \Omega / (GS)^3 \, kT \tag{2}$$

where  $D_b$  is the boundary diffusion coefficient, W is the boundary width, and GS is the average grain size. The grain size dependence and the numerical constant differ from Herring's assumptions.

Possibilities for crystal size enhancement were shown by Birringer<sup>5</sup> where grain boundary diffusivity *D* for a 8-nm crystal was enhanced by a factor of 10<sup>19</sup> when compared with the lattice diffusion. Such enhancement may be the result of low impurity concentration in the nanocrystalline boundaries as opposed to that of common polycrystals; it could also be rapid diffusion that moves along the connected boundary triple junctions. The earlier enhancement could be the result of the different boundary structures than those of polycrystals.

In the grain's geometry, two restrictions apply. The stress distribution must vary continuously, and the nanocrystalline matrix must continue to expand. As shown in Fig. 1, the atoms relax from the ideal lattice sites given by a hard-sphere model.<sup>5</sup> The relaxation

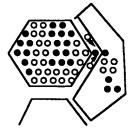


Fig. 1 Nanocrystalline cross section. Different interatomic spacings are shown with arrows.

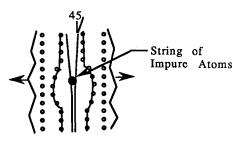


Fig. 2 Boundary interface between slightly tilted crystal grains with primitive lattice structure. The string of impurity is perpendicular to the plane of the figure.

involves the atoms at the boundary and extends several layers into the lattice of the adjacent crystals. Lattice dislocations are known to interact with grain boundaries. Thus, homogeneous materials should be attractive, due to the weakening of adjacent interatomic bonds by boundaries, and induce high-mobile defects. Boundaries of crystallites commonly accumulate dislocations against those escaping the rigid surrounding grains. This activity causes the

Using Fick's law and either Eq. (5) or Eq. (6) for the flux, one finds that the maximum gradient at the surface is

$$\left(\frac{dC}{R\,d\theta}\right)_{\theta=45} = \left(\frac{J_{\theta}}{D_{\nu}W2\pi R\sin\theta}\right)_{\theta=45} = 0.414 \frac{BR}{D_{\nu}W} \tag{7}$$

Hence, the proportionality constant N is

$$N = \frac{\left[\frac{dC}{(Rd\theta)}\right]_{\theta = 45}}{\Delta C/(R\pi/2)} = \frac{0.414 (BR/D_v W)}{\left[BR/(D_v W \pi/2)\right] \left\{ \int_0^{45} \left[ (1 - \cos\theta) d\theta / \sin\theta \right] + \int_{45}^{90} \left[ (1 - \sin\theta) d\theta / \sin\theta \right] \right\}} = 2.56$$
(8)

matrix to grow near the boundaries, under the condition that the interaction of lattice dislocations with boundaries is greater than the lattice friction forces.<sup>3</sup>

In a spherical model, following Coble's assumptions, each grain face has a uniform normal tensile component, and suitable areas for the vacancy source and sink can be selected, assuming the generation and annihilation rates are uniform.

## **Model Formulation**

As shown in earlier experimental work,<sup>3</sup> the presence of an active dislocation source may represent either localized plastic flow energy concentrations or, as they appear in nanocrystallines, shear bands. The source is formed by a string of precipitate impurity atoms or molecules collected at a grain boundary. Assuming that a slight angular misalignment exists between the adjacent lattices, with the addition of chain impurities, it can be taken that the tensile angle from the pole is 45 deg, as shown in Fig. 2.

The boundary interface<sup>3</sup> between two slightly tilted crystal grains with simple lattice structure is shown in Fig. 3. Once a load of compression and shear is applied, the impurities are "locked" in (Fig. 3a). The loading conditions will make the host atoms adjacent to the impurities align with the impurity chain (Figs. 3b and 3c). With the tensile loading in effect, the spherical crystals tend to overlap and make the 45 deg offset tilt from the main axis, consequently capturing impurities which in turn become one impurity chain again (Fig. 3d). In order for an angle formation greater than 45 deg to occur, the impurity chain must be assumed nonexistent. With the dislocations moving from the source region, the loading is then relaxed, the host atoms along the impurity chain increase, and the process repeats itself (Fig. 3e). The commencement of such movement that may determine the vacancy source along with Coble's assumptions and Eq. (2) will lead to the creep model.

Thus, for a nanocrystal where the boundary of rotational symmetry among equal areas lies at 45 deg below the pole (on a hemisphere), the concentration gradient at  $\theta = 45$  deg boundary is needed. Applying Fick's law for steady-state creep,<sup>2</sup> thus the spacings per second J are

$$J = D_{\nu} N[\Delta C / (R\pi/2)] W 2\pi R \sin \theta \tag{3}$$

where  $[\Delta C/(R\pi/2)]$  is the average gradient. For  $\theta=45$  deg in Eq. (3) the flux is a maximum. In Eq. (3) R is the radius of the spherical analog,  $D_v$  is the diffusion coefficient of vacancies in the boundary, N is the proportionality constant between the average gradient and the maximum gradient  $[1/R(dC/d\theta)_{\theta=45}]$ , W is the effective boundary width, and  $(W2\pi R \sin 45)$  is the diffusional cross-sectional area. Evaluation of N and its sensitivity to the spacing source and site distribution is needed.

The flux increases linearly with area A away from the pole<sup>2</sup>:

$$dJ = B dA = B2\pi R^2 \sin \theta d\theta \tag{4}$$

where B is in vacancies per unit area per second. If it is required that the flux be zero at the pole  $(\theta = 0)$  and at the equator  $(\theta = 90)$  deg) and a maximum at  $\theta = 45$  deg, it could be found that

$$J_{\theta} = 2\pi B R^2 (1 - \cos \theta) \qquad 0 \le \theta \le 45 \text{ deg}$$
 (5)

$$J_{\theta} = 2\pi B R^2 (1 - \sin \theta) \qquad 45 \le \theta \le 90 \text{ deg} \tag{6}$$

which is very close to the value Coble calculated. Thus, using Eq. (8) and Eq. (3) for a boundary diffusion model the diffusion flux is

$$J = 7.23 D_{\nu}(\Delta C)W$$
 vacancies/cm s (9)

Using Eq. (9) for the vacancies diffusing from a uniform source of area  $\pi R^2$ , the volume change is<sup>2</sup>

$$Ja_0^3 = \pi R^2 (dR/dt) = 7.23 D_{\nu}(\Delta C) W a_0^3$$
 (10)

and the creep rate is

$$\dot{e} = 1/R (dR/dt) = 7.23 D_{\nu} (\Delta C) W a_0^3 / (\pi R^3)$$
 (11)

The shape does not change only by diffusion but also because of the inherent shear stress relaxation at the grain boundaries. The shape change is shown in Fig 3. This relaxation at the boundaries results in an altered creep rate for the boundary diffusion model that is shown to be  $\dot{e}_{\text{relaxed}} = 2.5 \dot{e}_{\text{unrelaxed}}$ . Using the coefficient for atoms in the boundaries  $D_b = D_v C_0 a_0^3$ , and calling the grain size (GS) = 2R, along with Eq. (1), one finds that the creep rate is

$$\dot{e} = 145 \sigma D_b W a_0^3 / [(GS)^3 kT]$$
 (12)

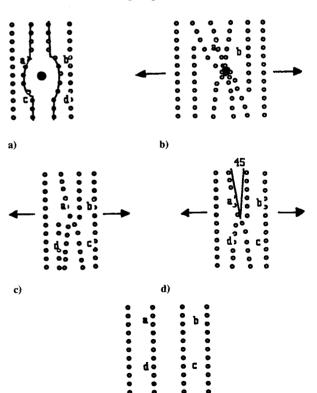


Fig. 3 Schematic configuration of crystal grains: a) lattices under compressive and shear loads, b) vertical impurity causes overlapping of the chains after application of forces, c) lattice rearrangement and creation of a pair of dislocations by the applied stress, d) angle offset in the mode under stresses, and e) starting and ending stage.

e)

which is very similar to the rate by Coble, compares well with the lattice diffusion model by Herring, and agrees with experimental results.<sup>7</sup>

If the grain size is approximately an order of magnitude larger than the thickness within which diffusion is enhanced, Eq. (12) provides very good agreement with experimental results.<sup>7</sup> If the grain sizes are on the order of the apparent widths of boundaries deducted from measurements, the lattice model<sup>1</sup> should be used.

### **Conclusions**

A boundary diffusion controlled creep model for nanocrystals based on existing models and theories has been developed. In the spherical analog for the vacancy source, the boundary of rotational symmetry between equal areas was taken to be 45 deg below the nanocrystalline pole (see Fig. 3). It is at 45 deg that the maximum flux exists. The proportionality constant N and the creep rate compare well with other studies. It is agreed that creep control is more successful by boundary diffusion when it takes place at larger grain sizes and smaller interfacial components.

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# Elastodynamic Characteristics of Hollow Cantilever Beams Containing an Electrorheological Fluid: Experimental Results

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# Introduction

THE vibration control of flexible structures with variable disturbances in different frequencies becomes more and more important because of the demand for higher positional accuracy and more stable manipulation. To achieve this goal, the resonance of the structures must be avoided in all cases. For traditional structures, the frequencies of the disturbances

must be known and guessed before the design of the structures to avoid the excitations imposed in the neighborhood of the structural natural frequencies. For variable excitations, this methodology cannot be adopted. However, smart material structures that have their own sensing, actuating, tuning, controlling, and computational capabilities can be adjusted to variable disturbances. Typically, smart materials and structures¹ are employed to control the static and elastodynamic responses of distributed parameter systems. This may be accomplished by controlling the mass distribution, stiffness characteristics, and energy dissipation characteristics of the structure. Recently, significant progress has been made in the development of smart materials and structures that incorporate piezoelectric materials,² shape memory alloy,³ and electrorheological (ER) fluids.⁴

This paper reports on a proof-of-concept experimental investigation focused on evaluating the elastodynamic characteristics of hollow cantilever beams filled with a hydrous-based ER fluid consisting of cornstarch and silicone oil. The beams are considered to be uniform viscoelastic materials and modeled as a viscously damped harmonic oscillator. Electric-field-dependent natural frequencies, loss factors, and complex moduli are evaluated and compared among three different beams: two types of different volume fraction of the ER fluid and one type of different particle concentration of the ER fluid by weight. It is also shown that, by tailoring the electric field, structural resonances can be avoided to provide the feasibility of active vibration control applications under unstructured environmental conditions.

## **Experimental Apparatus and Procedures**

The structure of the proposed beam specimen shown in Fig. 1 consists of two faceplates, rubber and acrylic. The faceplates can act as electrodes, and the rubber serves as a seal to hold the integrity of the specimen. The specifications of the ER fluid and three specimens used in this study are presented in Table 1. The cantilever specimen was clamped in a fixture that was bolted down on top of the shaker head as shown in Fig. 2. A noncontacting probe was mounted near the tip of the specimen to pick up the vibration responses. In forced vibration, the damped natural frequencies with respect to the electric field were measured through the fast Fourier transform (FFT) analyzer by sweeping the sine wave from 0 to 100 Hz.

In free vibration, by assuming that the first mode is dominant and that the material is uniform and viscoelastic, we modeled the specimen as a single-degree-of-freedom damped free harmonic oscillator. Thus, for free oscillation, the governing equation of motion can be written as follows:

$$m_{\rm eff}\ddot{x}(t) + c\dot{x}(t) + k_{\rm eff}x(t) = 0 \tag{1}$$

where x(t) is the displacement,  $m_{\rm eff}$  is the effective mass, c is the damping constant, and  $k_{\rm eff}$  is the effective spring constant. The effective stiffness  $k_{\rm eff}$  (=  $3E_RI/L^3$ ) is the deflection at the end of the beam due to the unit load, and the effective mass  $m_{\rm eff}$  (= 0.236 m) is the concentrated mass located at the end of

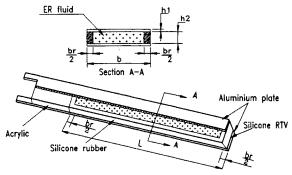


Fig. 1 Schematic diagram of the beam specimen.

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